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Comparison of Four Distribution Functions for Fitting Diameter in Second Rotation *Tectona grandis* Linn. f. Plantations in Eda Forest Reserve, Nigeria

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Abstract

Tree diameter is a very important variable in forestry; its accurate description is vital to planning if the forest is to be managed in a sustainable manner. Probability density functions are widely used for characterizing tree diameter. However, there is limited information on the diameter distribution for most of plantation in the country especially in the study area. Therefore, this research assesses the performance and suitability of beta, 3-parameter gamma, 3-parameter lognormal and 3-parameter Weibull distribution functions for fitting diameter at breast height (Dbh) of the second rotation of *Tectona grandis* Linn.F plantation in Eda Forest Reserve, Nigeria. Sixteen (16) temporary sample plots, (each 20 m x 20 m) were randomly selected across four second rotation age series (7, 8, 10 and 11years), whose originalages were 24, 31, 32 and 37 respectively. The distribution functions were fitted using maximum likelihood estimators. The performance of each function was tested using the Kolmogorov-Smirnov (K-S) statistic, Anderson Darling test, Chi-square, mean absolute error (MAE) and mean square error (MSE). The test results revealedthat 3-Parameter Weibull distribution was the most suitable for characterising Dbh. Weibull distribution had the least Kolmogorov-Smirnov statistic (0.0453), Anderson Darling test (1.231), Chi-square (24.9700), mean absolute error (0.0964) and mean square error (2.1688). Therefore, 3-parameter Weibull distribution is the best and recommended for fitting Dbh of second rotation *T. grandis* species in the study area for sustainable teak timber management.

Keywords: Diameter at breast height; Diameter distribution; Second rotation plantation; Probability density functions; *Tectona grandis*

Introduction

Sustainable forest management requires information about both current and future conditions of a growing stock. Such information is vital for valuation and decision-making. Diameter at breast height (Dbh) is an important tree growth variable and easily measured with simple instruments and widely used in forest inventories. Earlier studies on tree diameter have shown that information about Dbh can be used in estimation of various tree and stand variables; volume (Akindele and LeMay, 2006), height (Castedo-Dorado, 2005), crown dimensions (Ezenwenyi and Chukwu, 2017).Stand yields have also been predicted based on the assumption that diameter distribution of a stand can be characterized by a probability density function (Poudel, 2011). Hence, Dbh have been well characterized by means of distribution functions (Bailey and Dell 1973). Tree diameter distribution explores the structure of a forest and aids the forest manager in developing management plan for a plantation that ensures sustainability (Rouvinen and Kuuluvainen, 2005).

In forestry, various distribution functions such as normal, gamma, lognormal, Johnson's SB, beta, and Weibull have been successfully used to describe tree diameter distributions of forest stands (Palahi *et al.,* 2007; Aigbe and Omokhua 2014; Ogana *et al*., 2015). Weibull distribution is often and widely used owing to its simplicity and flexibility and its parameters are easy to estimate (Bailey and Dell 1973; Nord-Larsen and Cao 2006; Gorgoso *et al*., 2007). In Nigeria, Weibull distribution has been used to fit diameters from both natural and plantation forest (Ajayi, 2013;

Ogana and Gorgoso-Varela, 2015; Ogana *et al.*, 2015,), yet little have been done in comparing this to other distribution functions in describing forest stands. However, there is no *a priori* biological basis for using the Weibull distribution or indeed, any other statistical function (Shiver 1988). Thus, the selection of the distribution function is entirely up to the researcher (Siipilehto and Mehtätalo, 2013). Furthermore, efforts have focused on the diameter distribution in plantations that have not been cropped; while the second rotation (*i.e.* re-growth after coppicing) plantation has been left unconsidered. If the second rotation plantations are to be effectively managed in a sustainable manner, there is a need to assess the diameter distribution of such plantation.

Over the years, Weibull distribution has gained high popularity for characterising tree diameter in Nigeria and other parts of the world. However, studies on Beta, Gamma and Lognormal distribution model are scares, Ogana *et al.* (2015) attributed the scarcity to complexity of computation in estimating the parameters of the other distribution functions. Previous studies (Ogana *et al.,* 2015; Ogana and Gorgoso-Varela 2015) on probability density functions have shown that the number of parameter of a function affects its performance. This study therefore seeks to explore and compare the relative suitability of beta, 3-parameter gamma, and 3-parameter lognormal and 3-parameter Weibull diameter distributions for describing Dbh of second rotation plantations of teak (*Tectona grandis* Linn. f.) in Eda Forest Reserve, Nigeria for sustainable timber management.

Methodology

Study Area

This study was carried out in the Eda forest reserve in Ekiti State, Nigeria. The forest reserve is divided into two sectors, namely, Eda I and Eda II, for plantation and natural forest, respectively. Eda I is situated at the northern part of Eda II. The entire forest reserve is located between Latitudes 7° 23' N and 7° 46' N and Longitudes 4° 47′ E and 5° 45' E (Alo *et al.,* 2014). Average elevation is 514 m above sea level. The climate is entirely tropical, with two distinct seasons: raining and dry seasons. The raining season spans from March to November while the dry season is from December to February. Annual temperature and average relative humidity are 26.5°C and 70%, respectively. Soil parent materials were formed from sedimentary rocks, mainly crystalline rocks of the undifferentiated basement complex of the pre-Cambrian series. The soil comprises of well-drained, mature, red, stony and gravely soil in the upper part of the sequences of tropical rainforest zone of Nigeria (Onyekwelu *et al.*, 2006).

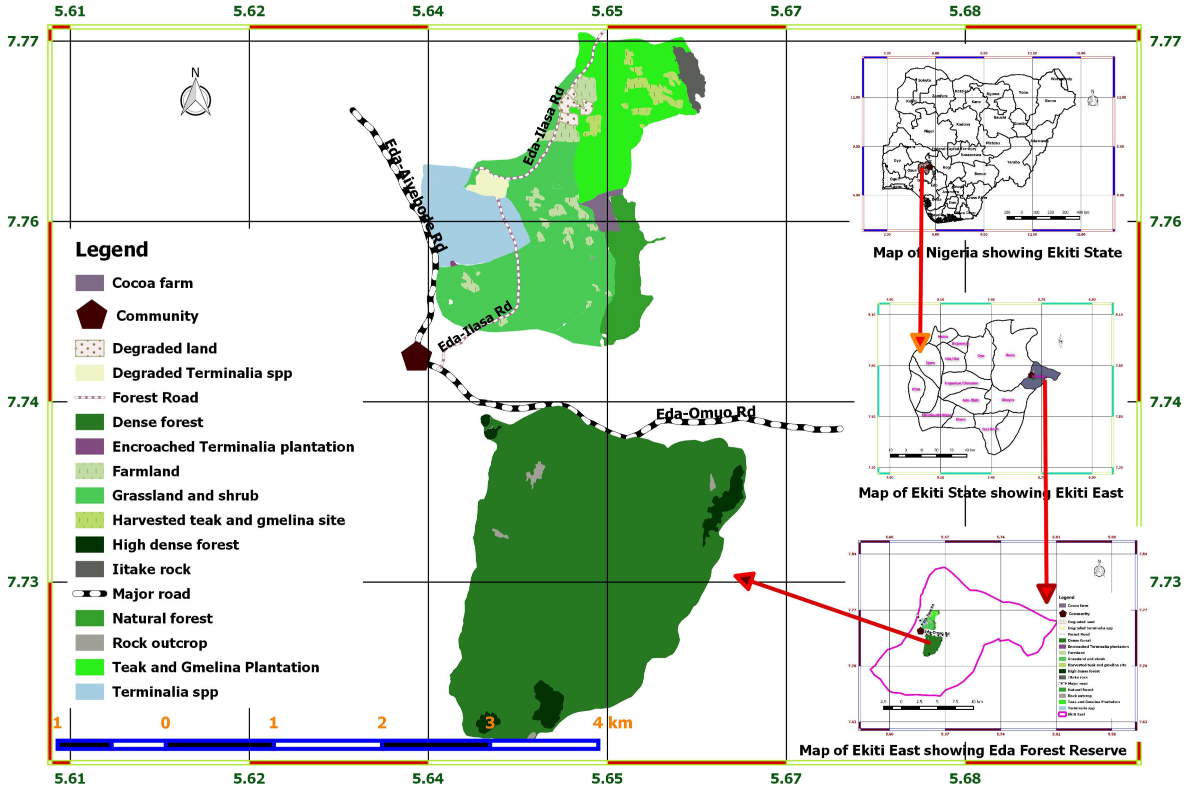

Figure 1: Eda Forest Reserve, Nigeria
Source: Alo *et al.,*(2015)

Sampling Procedure and Data Collection

This study was carried out in temporary sample plots (TSPs) of *Tectona grandis* stands of four rotation ages; 7, 8, 10 and 11years, whose original ages were 24, 31, 32 and 37, respectivelyfound in the study area. A stratified random sampling technique was employed for this study. The four-age series constitute the strata in the study area. Hence, simple random sampling technique was used in allocating sixteen (16) TSPs of 20 m x 20 m size in the stands (four plots per age stratum). A total number of six hundred and eighty (680) *T. grandis* species with Diameter at breast height (Dbh) ≥7.0 cm in the sixteen randomly selected sample plots.

Tree diameter

Dbh was recorded for all tree individuals with values of ≥ 7.0 cm. The point of the measurement was recorded from the uphill sides of the trees and on the inside of the lean for leaning trees. For trees with deformations at 1.3 m, the measurement was made at the sound point on the stem above the abnormality. Diameter measurements of trees were made using a metric diameter tape graduated in centimetres. During the measurement, loose bark, climbers and epiphytes were lifted above the measuring tape.

Data Analysis

Diameter Fitting Procedureand Assessment

In this study, Dbh data from each plot were grouped into 5.0 cm class intervals. Diameter distribution functions were fitted using the maximum likelihood method. Beta, Gamma (3p), Lognormal and Weibull distribution functions were used in characterizing the Dbh data of *T. grandis* trees enumerated in the study area.

Beta Distribution

The beta distribution function (Krishnamoorthy, 2006) is expressed as:

$$f(x) = \frac{1}{B(\alpha_1,\alpha_2)} \frac{(x-a)^{\alpha_1-1}(b-x)^{\alpha_2-1}}{(b-a)^{\alpha_1+\alpha_2-1}}$$

Equation [1]

Where

α_1 *and* α_2 are shape parameters (α_1, α_2>0),

a, b, are the limits of the distribution (a<b)

$B(\alpha_1,\alpha_2)$ is the beta function. It has the formula:

$$B(\alpha_1,\alpha_2) = \int_0^1 t^{\alpha_1-1}(1-t)^{\alpha_2-1}\, dt \; (\alpha_1,\alpha_2 > 0),$$

Equation [2]

Lognormal Distribution

The probability density function (pdf) of the three-parameter lognormal distribution (Aristizabal, 2012) is:

$$f(x;\mu,\sigma,\gamma) = \frac{1}{(x-\gamma)\sigma\sqrt{2\pi}} exp\left\{\frac{[ln(x-\gamma)-\mu]^2}{2\sigma^2}\right\}$$

Equation [3]

where x > γ ≥ 0 , − ∞ < μ < ∞ , σ > 0 , and γ is the threshold parameter or location parameter that defines the point where the support set of the distribution begins; μ is the scale parameter that stretch or shrink the distribution and σ is the shape parameter that affects the shape of the distribution. If X is a random variable that has a three-parameter log-normal probability distribution, then Y = ln(X − γ) has a normal distribution with mean μ and variance σ^2. The two-parameter lognormal distribution is a special case of the three-parameter lognormal distribution when γ = 0.

The Gamma Distribution

The 3-parameter gamma distribution function (Krishnamoorthy, 2006) is expressed as:

$$f(x) = \frac{(x-\gamma)^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e\left(-\left(\frac{x-\gamma}{\beta}\right)\right)$$

Equation [4]

Where:

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1}e^{-t}\, dt \qquad (\alpha > 0)$$

α is the shape parameter (α > 0), β is the scale parameter (β > 0), γ is the location parameter (γ ≡ 0, for a distribution with two parameters); *x* = diameter (Dbh).

The Weibull Distribution

The 3-parameters Weibull distribution (Weibull 1951) was used for this study. It is expressed as:

$$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} exp\left[-\left(\frac{x-a}{b}\right)^{c}\right]$$

Equation [5]

The Weibull cumulative distribution function (CDF) is obtained by the integration of the above function. It is expressed as:

$$F(x) = \int_0^x \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] dx$$

Equation [6]

$$F(x) = 1 - exp\left[-\left(\frac{x-a}{b}\right)^{c}\right]$$

Equation [7]

Where: *F(x)* is the Weibull cumulative distribution function; *x* is tree diameter to be measured, a, b and c are the location, scale and shape parameters of the distribution respectively.

Criteria for comparing Distributions

The Kolmogorov Smirnov, Anderson Darling, Chi-Squared, mean absolute error (MAE) and mean square error (MSE) were computed for each fit in mean relative frequency of trees per one for all Dbh classes, and used as goodness of fit measures for this study: Hence, method with the least values of these measures was selected. They are mathematically expressed as:

Kolmogorov Smirnov (K-S) Test:

$$D_n = Supx|F(x_i) - F_0(x_i)|$$

Equation [8]

Where:

Supx is the supremum value,

$F(x_i)$is the cumulative frequency distribution observed for the sample x_i(i = 1, 2…, n)

$F_0(x_i)$is the probability of the theoretical cumulative frequency distribution. Dbh classes of 5cm intervals were selected.

Anderson Darling Test:

$$A^2 = -N - S$$

Equation [9]

Where:

$$S = \sum_{i=1}^{N} \frac{(2i-1)}{N} [\ln F(Y_i) + \ln (1 - F(Y_{N+1-i}))]$$

Equation [10]

Where: F is the cumulative distribution function of the specified distribution and $Y_i$are the ordered data.

Chi-squared

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

Equation [11]

Where: O_i is the observed frequency for bin $i$and $E_i$is the expected frequency for bin *i*. The expected frequency is calculated by

$$E_i = N(F(Y_u) - F(Y_l))$$

Equation [12]

Where: F is the cumulative distribution function for the distribution being tested, Y_u is the upper limit for class *i*, $Y_l$is the lower limit for class *I*, and$N$ is the sample size.

Mean Absolute Error (MAE):

$$MAE = \frac{\sum_{i=1}^{N} |Y_i - \hat{Y}_i|}{N}$$

Equation [13]

Mean Square Error (MSE):

$$MSE = \frac{\sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2}{N}$$

Equation [14]

Where:$Y_i$is the observed value,$\hat{Y}_i$, is the theoretical value predicted by the model and N is the number of data points

Results

The summary statistics for the tree stem diameter measured from the second rotation plantation of *T. grandis* species investigated (Table 1) showed that the distribution of diameter at breast height (Dbh) ranged from 7.21cm to 37.60 cm in all age series. The mean values with corresponding standard error (SE) for the Dbh for the four ages studied were presented in the Table 1.

Table 1: Summary statistics of data used for the study

Age (years)		Statistics			
Original	Rotation	Minimum	Maximum	Mean ± Std. error	No. of trees
24	7	7.60	26.22	14.23 ± 0.23	230
31	8	7.80	29.31	14.37 ± 0.38	133
32	10	8.20	28.10	14.73 ± 0.29	173
37	11	7.21	37.60	15.68 ± 0.44	144

Graphical analyses (Figures 1-5) of the observed numbers of trees and the predicted frequency by the four distribution functions displayedrelatively of a plantation forest. The graph revealed that, the expected frequency of trees produced by Beta, Gamma (3p), Lognormal (3p) and Weibull (3p) distribution functions fitted with maximum likelihood method exhibited slight variation with the observed diameter distribution. Thus, larger proportion of trees are found at the middle diameter classes with decreasing frequency at both sides; given rise to the nearly Bell-shaped structure as shown in Figures 1 to 5.

The result of the goodness of fit on age group basis (Table 1) showed that; on the basis of Kolmogorov Smirnov, Anderson Darling, Chi-Squared MAE and MSE statistics, Weibull (3P) distributions ranked 1st in three different ages (7, 10 and 11 years) and second in age 8. And lognormal (3P) model ranked fourth in ages 7 and 8 and third in age 11. The individual values of Kolmogorov Smirnov, Anderson Darling, Chi-Squared MAE and MSE for each of the distributions were presented in Table 2.

Table 2: Goodness of fit result for diameter distribution of each age group

Age (years)		Distribution	Goodness of Fit – Summary					Rank
Org.	Rot.		K-S	A^2	χ^2	MAE	MSE	
24	7	Beta	0.0436	0.4433	4.0479	0.0872	0.6135	2
		Gamma (3P)	0.0599	0.7030	8.4064	0.1269	1.2448	3
		Lognormal (3P)	0.0599	0.8180	8.5148	0.1276	1.2247	4
		Weibull (3P)	0.0465	0.5232	2.9614	0.0891	0.5960	1
31	8	Beta	0.0459	0.2822	2.1460	0.0872	0.2658	1
		Gamma (3P)	0.0668	0.6898	2.7692	0.1269	0.7435	3
		Lognormal (3P)	0.0634	0.7307	5.9836	0.1276	0.7388	4
		Weibull (3P)	0.0585	0.4632	4.5360	0.0891	0.4357	2
32	10	Beta	0.0688	0.8032	9.2275	0.0872	1.4602	4
		Gamma (3P)	0.0670	0.9303	6.3837	0.1269	1.3050	3
		Lognormal (3P)	0.0656	0.8674	6.7247	0.1276	0.9603	2
		Weibull (3P)	0.0652	0.8104	5.4501	0.0891	1.2214	1
37	11	Beta	0.0883	0.9549	21.580	0.0872	0.6591	2
		Gamma (3P)	0.0976	1.0888	20.223	0.1269	0.7253	4
		Lognormal (3P)	0.0948	1.0601	12.985	0.1276	0.5667	3
		Weibull (3P)	0.0881	0.8877	16.912	0.0891	0.5338	1

Where: K-S = Kolmogorov Smirnov test statistics, A^2 = Anderson Darling test statistics, χ^2= Chi square test, MAE= mean absolute error and MSE= means square error.

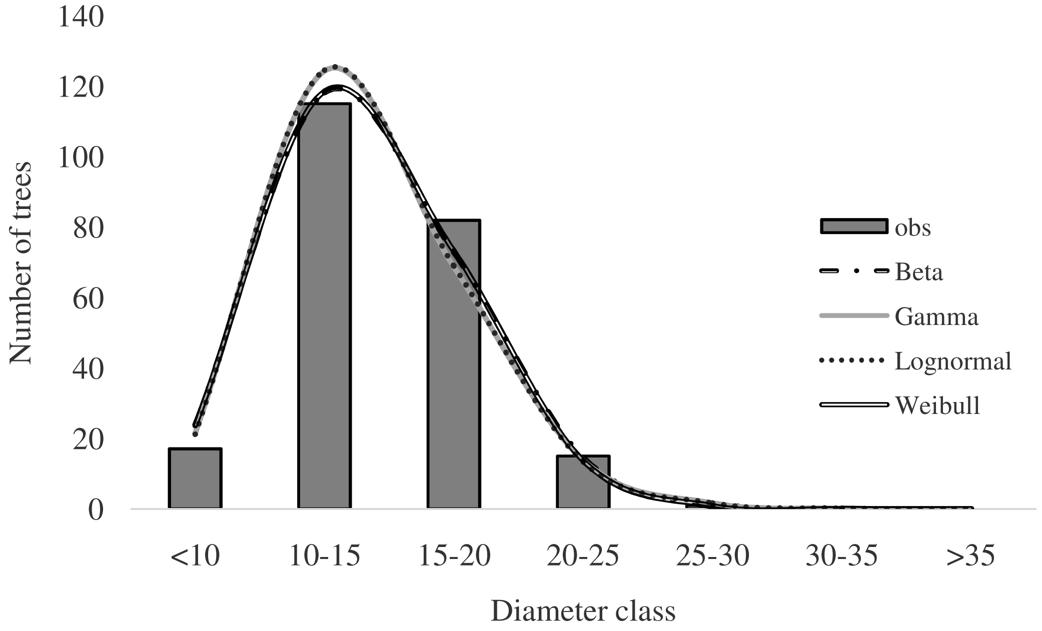

Figure 1: Observed and the four (4) fitted diameter distribution using maximum likelihood method for age 7

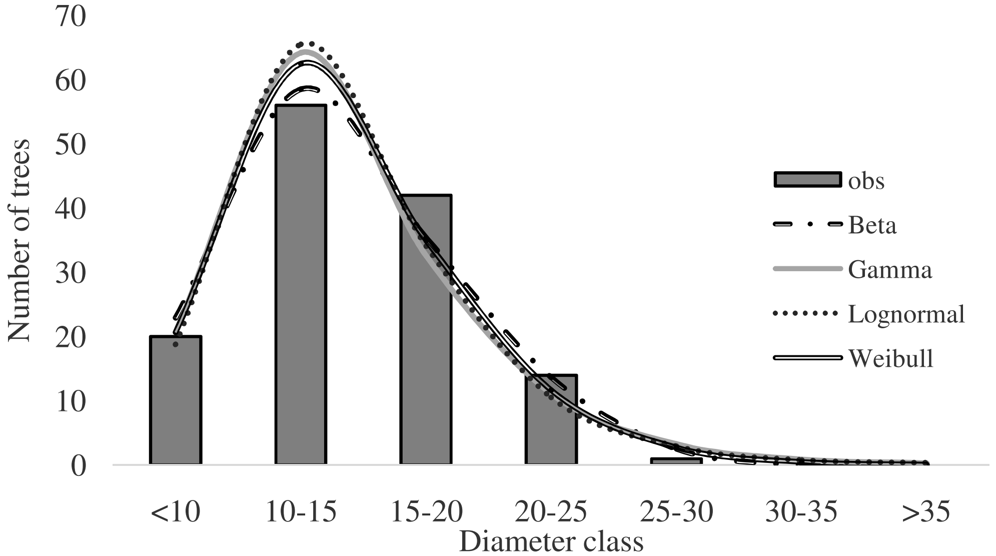

Figure 2: Observed and the four (4) fitted diameter distribution using maximum likelihood method for age 8

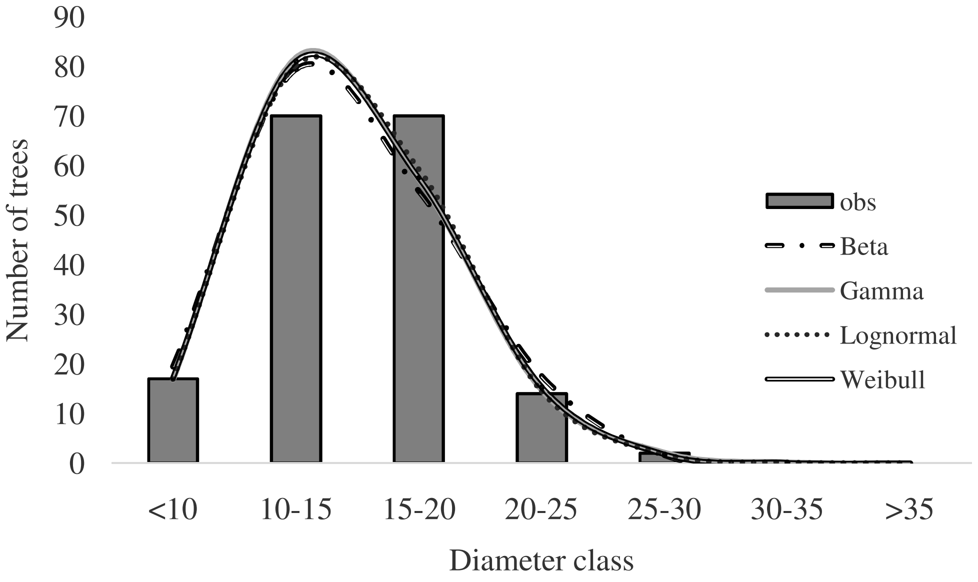

Figure 3: Observed and the four (4) fitted diameter distribution using maximum likelihood method for age 10

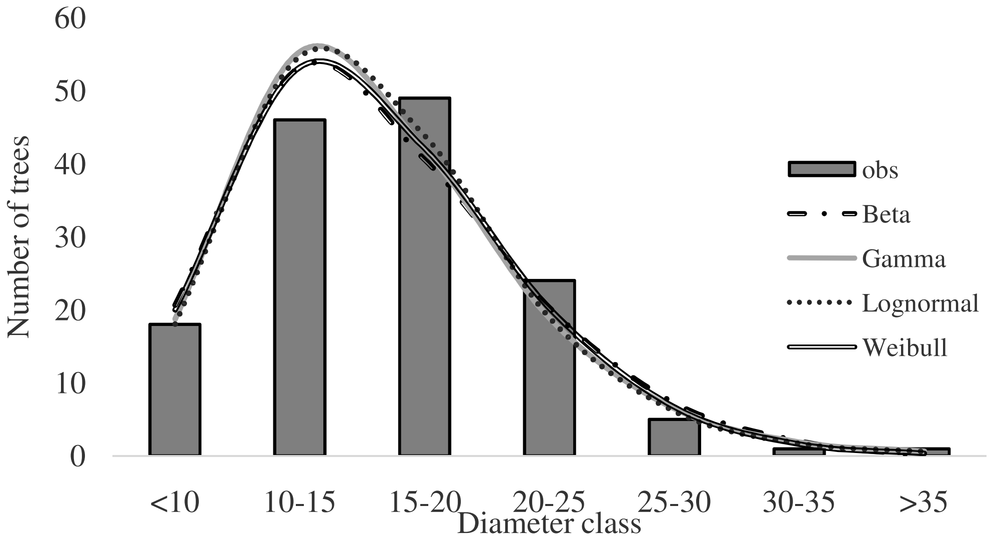

Figure 4: Observed and the four (4) fitted diameter distribution using maximum likelihood method for age 11

Furthermore, the goodness of fit result of diameter distributions for the pooled Dbh data were presented in Table3. Weibull (3P) had the least values of Kolmogorov Smirnov, Anderson Darling, Chi-Squared MAE and MSE; 0.0453, 1.231, 24.9700, 0.0964 and 2.1688 respectively, followed by Beta distribution while Gamma (3P) distribution highest of Kolmogorov Smirnov statistics (0.0608), Anderson Darling test statistics (2.1936), MAE (0.1383) and MSE (4.3628). Furthermore, the result also revealed that Lognormal (3P) distribution with the highest Chi-Squared value of 56.1130 as shown in Table 3.

Table 3: Goodness of fit result for diameter distribution of the pooled data

Distribution	Goodness of Fit - Summary					Rank
	K-S	A-D	χ^2	MAE	MSE	
Beta	0.0545	1.6230	26.3240	0.1207	3.4135	2
Gamma (3P)	0.0608	2.1936	45.1010	0.1383	4.3628	4
Lognormal (3P)	0.0601	2.4530	56.1130	0.1331	4.0253	3
Weibull (3P)	0.0453	1.2311	24.9700	0.0964	2.1688	1

Where: K-S = Kolmogorov Smirnov test statistics, A^2 = Anderson Darling test statistics, χ^2= Chi square test, MAE= mean absolute error and MSE= means square error.

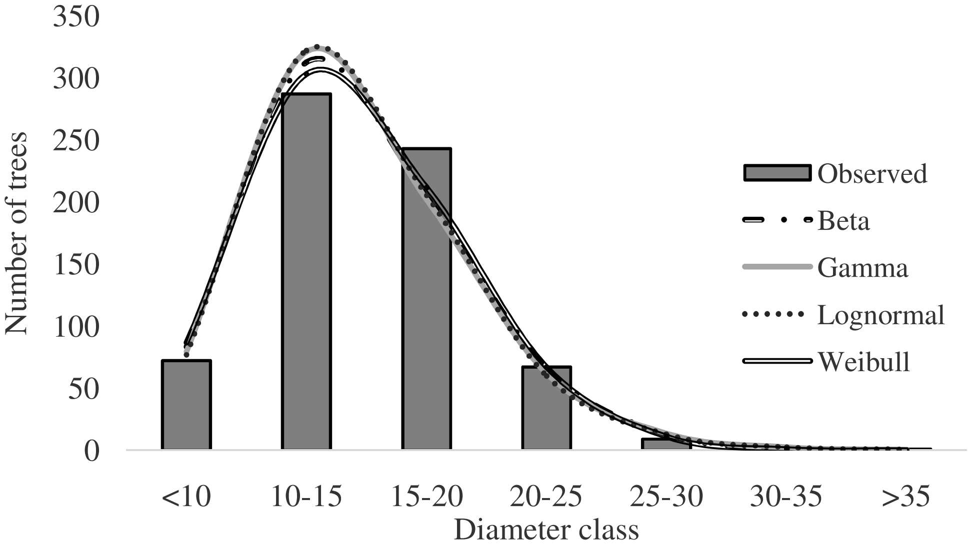

Figure 5: Observed and the four (4) fitted diameter distribution using maximum likelihood method for the pooled data

Discussion

Diameter at breast height (Dbh) is one of the most important growth variables in forestry. Tree volume, value, conversion costs, and product specifications are dependent on Dbh. Hence, most stand properties can as well characterize with diameter distributions (Bailey and Dell 1973).In this study, the fit performance of beta, Gamma (3p), Lognormal (3p) and Weibull (3p) distribution functions for describing the Dbh of Second Rotation of *T. grandis* plantation in the study area were assess/adjudge using the mean values of Kolmogorov Smirnov (K-S) statistic, Anderson Darling, Chi-square, mean absolute error (MAE) and mean square error (MSE). The results indicated that there were little variations in the four distribution functions considered in this study.

Graphical analysis result indicated that there were more trees in the lower diameter class than the upper diameter class. This could be due to the fact that the second rotation plantation age, ranges between 7 to 11 years, and therefore were relatively young. However, the non-perfect Bell-shaped structures displayed by graphs might also be because the stem diameters considered were from a second rotation plantation. The result for all evaluation statistics revealed that Weibull (3P) distribution performed slightly better than Beta distribution and was more consistent than the other distribution functions considered in this study both on age basis and for the pooled data. Hence, Weibull (3P) distribution was selected as the best model for the study area. This observation was in agreement with the research of Aigbe and Omokhua (2014) who found Weibull (3P) was more flexible than Beta, Burr 4P, Gamma 3P, Johnson SB, Lognormal distribution functions when tested with Kolmogorov Simonov and Chi-Square in modelling diameter distribution of the tropical rainforest in Oban Forest Reserve, Nigeria. Likewise, this study is also similar to the work of Ogana *et al.,* (2015) who reported that three-parameter Weibull distribution performed slightly better than the Beta and Gamma 3P distributions for Characterising tree diameter in Oluwa Forest Reserve, Ondo State, Nigeria. Similarly, Namiranian (1990) and Mataji *et al.,* (2000) reported that Weibull and Beta distributions were the most appropriate in fitting diameter at breast height data in Gorazbon district of Kheyroudkenar forest in Noshahr.

However, the good performance and consistency of Weibull distribution in this study, could be as a result of its ability to fit a range of distributions ranging from the reversed J-shaped through left skewed and symmetrical to right-skewed(Bailey and Dell, 1973;Shifley and Lentz, 1985). Furthermore, Gamma (3P) provided the worst fit to the second rotation Dbh data in the study area; with relatively large values for the goodness-of-fit statistics. This implies that the Gamma (3P) distribution was less appropriate for determining the structure of the second rotation plantation forest studied. This was in contradiction to the observations of Mohammad-Alizadeh *et al.* (2009) in Gorazbon district who concluded that the gamma distribution had the greater ability to determine the diameter at breast height distribution.

Conclusions

It is advantageous in sustainable forest management to use the appropriate statistical distributions in predicting and/or describing the condition of a forest stand. Beta, Gamma (3p), Lognormal (3p) and Weibull (3p) distribution functions can be used for describing Dbh data of *T. grandis* plantation in Eda Forest Reserve, Nigeria. Hence, 3-parameter Weibull distribution using maximum likelihood estimators gave an overall best description of the stem diameter for the second rotation *Tectona grandis* species in the study area. Therefore, three-parameter Weibull distribution using maximum likelihood estimators is hereby recommended for diameter distribution in similar ecosystem for second rotation *T. grandis* species.

References

Aigbe, H. I. and Omokhua, G. E. (2014): Modeling Diameter Distribution of the Tropical Rainforest in Oban Forest Reserve, *Journal of Environment and Ecology.* 5(2): 130-143.

Ajayi S. (2013): Diameter distribution for *Gmelina arborea* plantations in Ukpon River Forest Reserve, Cross River State, Nigeria, AFRREV STECH.; 2(1): 64-82.

Akindele S.O. and V.M. LeMay, (2006): Development of tree volume equations for common timber species in the tropical rain forest area of Nigeria. *Journal of Forest Ecology and Management.* Vol. 226. pp 41–48.

Alo A. A., Akindele, S. O. and. Onyekwelu J. C. (2015): Development of Land Use Map of Eda Forest Reserve in Ekiti State, Nigeria. *Journal of Applied Tropical Agriculture,* Nigeria, Vol. 20. No. 1. Pg. 19 – 23.

Alo, A. A., Akindele S. O. and Onyekwelu, J. C. (2014): Development of Information System for Forest Reserves in Ekiti State, Nigeria. *International Journal of Research in Agricultural Sciences,*1(6): 373-378.

Aristizabal, R. J. (2012): Estimating the Parameters of the Three-Parameter Lognormal Distribution, FIU Electronic Theses and Dissertations, Florida International University. Miami, Florida, pp. 1-46

Bailey, R. L. and Dell, T. R. (1973): Quantifying diameter distributions with the Weibull function, *For. Sci.* 1973; 19:97-104.

Castedo-Dorado, F., Barrio M., Parresol, B.R., Álvarez, J. G. (2005):A Stochastic Height-Diameter Model for Maritime Pine Ecoregions in Galicia (Northwestern Spain). *Ann. For. Sci.*.62: 455-465.

Ezenwenyi, J. U. and Chukwu, O. (2017): Effects of slenderness coefficient in crown area prediction for *Tectona grandis* Linn. f. in Omo Forest Reserve, Nigeria. *Current Life Science,* 3(4): 65-71.

Gorgoso, J. J., Alvarez-Gonzalez, J. G., Rojo, A. andGrandas-arias, J. A. (2007): Modelling diameter distributions of *Betulaalba* L.stands in northwest Spain with the two parameter Weibull function. Invest. *Agrar.:Sist. Recur. For.* 16(2): 113-123.

Krishnamoorthy, K. (2006): Handbook of statistical distributions with applications. London: Taylor and Francis, 2006. 346 p. (Statistics, a series of textbooks & monographs, 188p)

Mataji, A, Hojati, M., and Namiranian, M. (2000): The study of quantity distribution in diameter levels in natural forests by using probability distribution. *Iran's natural resources magazine, 53*(2): 165-171.

Mohammad-Alizadeh, K. H., Zobeiri, M., Nammiranian M., A., Marvie-Mohadjer M.R. (2009): Fitting of diameter distribution using some statistical models (distributions) (Case study: Khyroudkenar forest, Noshahr*). Iranian Journal of Forest and Poplar Research*, 17(1): 116 – 124.

Namiranian, M. (1990): The application of probability theories in determining trees distribution in different diameter levels. *Iranian Journal natural resource*, 44:93-108.

Nord–Larson, T. and Cao, Q.V. (2006): A diameter distribution model for even-aged beech in Denmark. *Forest Ecology and Management*, 231: 218-225.

Ogana, F. N. and Gorgoso-Varela, J. J. (2015): Comparison of Estimation Methods for Fitting Weibull Distribution to the Natural Stand of Oluwa Forest Reserve, Ondo State, Nigeria. *Journal of Research in Forestry, Wildlife and Environment*. 7(2): 81-90.

Ogana, F.N., Osho, J.S.A and Gorgoso-Varela, J.J. (2015): Comparison of Beta, Gamma Weibull Distributions for Characterising Tree Diameter in Oluwa Forest Reserve, Ondo State, Nigeria, *Journal of Natural Sciences Research,*5(4): 28-36.

Onyekwelu, J.C, Mosandl, R. and Stimm, B. (2006): Productivity, site evaluation and state of nutrition of *Gmelina arborea*plantations in tropical rainforest zone in South-western Nigeria. *Forest Ecology and Management*, 229: 214–227.

Palahi M, Pukkala T, Blasco E, Trasobares A. (2007): Comparison of beta, Johnson's SB, Weibull and truncated Weibull functions for modeling the diameter distribution of forest stands in Catalonia (north-east of Spain). *European Journal of Forest Research.* 126: 563-571.

Peng, C., Zhang, L., and Liu, J. (2001): Developing and Validating Nonlinear Height-Diameter Models for Major Tree Species of Ontario's Boreal Forest. *North. J. Appl. For.* 1: 87-94.

Poudel, K.P. (2011): Evaluation of methods to predict Weibull parameters for characterizing diameter distributions, Electronic Thesis and Dissertation Collection, Louisiana State University. http://etd.lsu.edu/docs/available/etd-06302011133644/unrestricted/Poudel_Thesis.pdf. Assessed on: 03-April-2017

Rouvinen, S. andKuuluvainen, T. (2005): Tree diameter distributions in natural and managed old *Pinussylvestris*-dominated forests. *Forest Ecology and Management* 208: 45–61.

Shifley, S. and Lentz, L. (1985): Quick estimation of the three-parameter Weibull to describe tree size distributions. *Forest Ecology and Management* 13: 195–203.

Shiver, B. D. (1988): Sample sizes and estimation methods for the Weibull distribution for unthinned slash pine plantations diameter distributions. *For. Sci.* 34 (3): 809–814.

Siipilehto, J. and Mehtätalo, L. (2013): Parameter recovery vs. parameter prediction for the distribution validated for Scots pine stands in Finland. *Silva Fennica.* 47(4): 1-22.

Weibull, W. A. (1951): Statistical distribution functions of wide applicability. *J. Appl. Mech.* 18: 293-297